

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let f be of bounded variation on $[a, b]$. Let V be defined on $[a, b]$ as follows : $V(x) = V_f(a, x)$ if $a < x \leq b$, $V(a) = 0$. Then prove that

- (a) V is an increasing function on $[a, b]$.
- (b) $V - f$ is an increasing function on $[a, b]$.

17. Assume that $\alpha \uparrow$ on $[a, b]$, then prove that the following three statements are equivalent.

- (a) $f \in R(\alpha)$ on $[a, b]$.
- (b) f satisfies Riemann's condition with respect to α on $[a, b]$.
- (c) $\underline{I}(f, \alpha) = \overline{I}(f, \alpha)$.

18. State and prove the second fundamental theorem of integral calculus.

19. State and prove Abel's limit theorem.

20. State and prove Weierstrass M-Test.

APRIL/MAY 2025

23PMA12 — REAL ANALYSIS – I

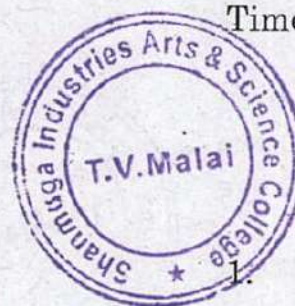
Time : Three hours

Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. What is Total Variation?
2. Define Partition of the interval.
3. State Riemann-Stieltjes Integral.
4. Define Riemann's condition.
5. State the sufficient conditions for the existence of Riemann-Stieltjes Integral.
6. State change of variable in a Riemann integral.
7. Define Double Series.
8. What is Power Series?
9. Define boundedly convergent sequence.
10. Define Mean Conveyance.



SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that if f is monotonic on $[a, b]$, then the set of discontinuities of f is countable.

Or

- (b) Show that the absolute convergence of $\sum a_n$ implies convergence.

12. (a) If $f \in R(\alpha)$ and $f \in R(\beta)$ on $[a, b]$, then $f \in R(c_1\alpha + c_2\beta)$ on $[a, b]$ (for any two constants c_1 and c_2) and we have

$$\int_a^b f d(c_1\alpha + c_2\beta) = c_1 \int_a^b f d\alpha + c_2 \int_a^b f d\beta \text{ - Prove.}$$

Or

- (b) Assume that α of on $[a, b]$. Then prove that $\underline{I}(f, \alpha) \leq \bar{I}(f, \alpha)$.

13. (a) If $f \in R$ and $g \in R$ on $[a, b]$, let

$$F(x) = \int_a^x f(t) dt, \quad G(x) = \int_a^x g(t) dt, \quad \text{if } x \in [a, b].$$

Then prove that F and G are continuous functions of bounded variation on $[a, b]$.

Also, $f \in R(G)$ and $g \in R(F)$ on $[a, b]$ and

$$\text{we have } \int_a^b f(x) g(x) dx = \int_a^b f(x) dG(x) = \int_a^b g(x) dF(x).$$

Or

- (b) Let α be of bounded variation on $[a, b]$ and assume that $f \in R(\alpha)$ on $[a, b]$. Then prove that $f \in R(\alpha)$ on every subinterval $[c, d]$ of $[a, b]$.

14. (a) Absolute convergence of $\sum u_n(1+an)$ implies convergence – prove.

Or

- (b) Assume that $\lim_{p, q \rightarrow \infty} f(p, q) = a$. For each find assume P_1 that the limit $\lim_{q \rightarrow \infty} f(p, q)$ exists.

Then prove that the limit $\lim_{p \rightarrow \infty} \left(\lim_{q \rightarrow \infty} f(p, q) \right)$ also exists and has the value a .

15. (a) Assume that $f_n \rightarrow f$ uniformly on S . If each f_n is continuous at a point C of S , then prove that the limit function f is also continuous at C .

Or

- (b) Assume that $\lim_{n \rightarrow \infty} f_n = f$ and $\lim_{n \rightarrow \infty} g_n = g$ on $[a, b]$. Define

$$h(x) = \int_a^x f(t) g(t) dt, \quad h_n(x) = \int_a^x f_n(t) g_n(t) dt, \quad \text{if } x \in [a, b].$$

Then show that $h_n \rightarrow h$ uniformly on $[a, b]$.

